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THE MATHEMATICS TEACHER

VOLUME XX NOVEMBER, 1927 NUMBER 7

THE TEACHING OF MATHEMATICS IN GERMANY SINCE THE WAR

BY FRITZ MALSCH

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TRANSLATED BY PROFESSOR RALPH BEATLEY

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The recent political changes which have been taking place in Germany have brought about—among other things—an eager desire for complete reorganization of the educational system of that country. The problems of the new education have occupied the attention of committees and congresses innumerable. Of these the most important is the Reichsschulkonferenz, a national committee, whose report has only recently been published.

Various movements for educational reform of one sort or another have been set on foot, and these have all had their ups and downs. A large number have languished after their first enthusiastic beginnings because of the excessive demands they made upon teachers or pupils. The ideas behind these new experiments have not been lost, however, but are exerting an ever wider influence upon educational thought. There is, moreover, a growing appreciation that the possibilities of the new are not unlimited, and that much of good resulted from the old system. Only a few representatives of the new movement in education can be mentioned here. In Munich, Kerschensteiner put his theories into practice in the common schools at a time when the new ideas could claim but few adherents anywhere else in Germany; Gaudig, headmaster of a large school in Leipzig, has been the leading exponent of a new method which, though widely criticized, is not without its good points; and Aloys Fischer, Eduard Spranger, and Theodor Litt have gained prom-

inence through their statements of the underlying theory, philosophy, implications, and scope of the new influences at work in education. The most significant of these new influences may be characterized by the catch words *psychological approach, learning by doing, project method,* and *education for life.*

These various movements culminated in the revision (in 1924–25) of the official program of instruction for secondary schools in Prussia, and the rest of Germany has followed suit on the whole, though by no means unanimously. But these new outlines and curricula have not put an end to discussion; rather they have stimulated it afresh. A few quotations from the introductory remarks which preface these new outlines will suffice to show their general tenor: "The final success of the reform movement in education hinges upon the possibility of constructing a working program which shall meet the insistent demands for unification of each curriculum. . . . The goal which German education should strive to attain is the cooperation of all the schools in one large common educational purpose, with each sort of school attaining this larger end in terms of its own special problems and consequent choice of subject matter. The increased emphasis on the development of an integrated personality as the proper aim of all educational effort, and the growing interest in national education, education for citizenship, education for art, and education for a philosophical appreciation of life and its problems, all demand a unification of each curriculum with respect to a large central purpose which is in marked contrast to the former emphasis on separate subjects for their own sakes. . . . Above all else, the instruction should be purposeful in a large sense. Consequently the teacher in selecting subject matter should consider not so much the demands of a given subject as the needs of his pupils, what skills they should practise, and how best they can develop independence of judgment, social mindedness, imagination, and incentive. To accomplish these ends the instruction should be organized as a social cooperative enterprise undertaken by the pupils under the guidance of the teacher, who shall adjust the work to the individual needs of the pupils and at the same time keep it in harmony with the larger aims of the school. . . . The particular methods employed should depend upon the nature of the subject matter and the intellectual maturity of the pupils."

In the following pages I shall discuss the effect of these larger trends upon the teaching of mathematics, and shall consider the present aims of instruction, the official course of study, and the contributions of leading teachers of mathematics. It will be impossible to give an exhaustive treatment, but this outline will serve at least to reveal the latest developments in this field and to indicate the forces which are now at work to bring to pass still further developments. For the benefit of those who may be interested in a more detailed treatment, I have appended to this article a bibliography which gives not only the most informative titles on the present educational movement in Germany, and on methods in mathematics in particular, but includes also the names of several textbooks which best exemplify these new ideas. I should like once again to emphasize that the movement to reform the teaching of mathematics was in origin quite independent of the recent educational reform movement in Germany, but that to be fully appreciated it must be considered now in relation to the larger movement.

Teachers and departments of mathematics were by no means unprepared for this reorganization of the secondary schools. For twenty years the problems of instruction in mathematics had been the subject of serious study throughout Germany. In 1908 these efforts spread beyond the national bounds with the foundation in Rome of the Internationale Mathematische Unterrichts-Kommission (I. M. U. K.). And before this, in 1905, the Society of German Naturalists and Doctors had by their discussions stimulated the development of new courses of study in mathematics and the sciences which are known collectively as "The Meran Syllabi." The German sub-committee of the I. M. U. K. produced from 1909 to 1916 a series of reports which treated in five volumes a variety of topics under the following general headings:

Vol. I. Secondary Schools in North Germany.
Vol. II. Secondary Schools in Middle and South Germany.
Vol. III. Various Aspects of Mathematical Instruction in Secondary Schools.
Vol. IV. Mathematics for Technical Schools.
Vol. V. The Teaching of Mathematics in the Lower Grades and Mathematics in Teacher Training Institutions.

Of these the third volume contains the material of greatest interest to teachers outside of Germany. The subjects it discusses are:

1. The development of the reform movement in mathematics.
2. Mathematics in textbooks on physics.
3. The teaching of mechanical drawing and descriptive geometry.
4. The mathematics of astronomy and elementary geodesy.
5. Problems in commerce in relation to the teaching of mathematics.
6. The history of mathematics and its bearing on instruction.
7. The philosophical aspects of mathematics.
8. Psychology and method in mathematics.
9. Mathematics in German universities.

The ninth is of special interest because of the personal notes and reminiscences of German mathematicians which it contains. Its masterly and exhaustive review of the teaching of mathematics in the universities captivates the reader through its very brilliance. Special mention should be made of the name of Felix Klein, the first president of the I. M. U. K., who labored so mightily for its success, as well as for its German offshoot. The testimonials which followed his death in June 1925 bear glowing witness to the contribution he made to the teaching of mathematics in Germany.

For a comprehensive statement of the principles and methods which have gradually evolved from the deliberations and discussions of those closest to the reform movement one should turn to Lietzmann's "Methodik des Mathematischen Unterrichts." From this book, already in its second edition, we see how far mathematics has emerged from its isolation of the last hundred years; how much more intimate its relations with economics, technology, government, politics, and general culture have become; how fundamentally it underlies all schooling for practical life because of its own basis in reality and fact; and how its connections with other fields of learning can be used to build up the student's philosophical understanding. This book affords, moreover, an exceptionally sound review of questions of method which arise in each branch of mathematics and its applications, and of current opinion concerning them.

In 1920 the Education Section of the Society of German Naturalists and Doctors passed a resolution to the effect that the

principles of "The Meran Syllabi" were still of basic importance in determining curricula and courses of study. It asked for even greater emphasis, however, on applications to technology and economics and on their relations to the community; and for greater emphasis also on space perception and functional thinking. That good logical training is withal necessary and still obtainable under this new order is obvious. A somewhat greater emphasis on geometric drawing and on the practical applications of mathematics is expected in the technical high schools, with more intensive treatment of analytic geometry and the differential and integral calculus. The Syllabi say, "The experiences of the World War have but intensified the age-old conviction that mathematics possesses an essential reality for which the exact sciences are forever indebted. This reality has accordingly been given greater stress in recent courses of study in the form of more practical applications, and functional thinking has been invoked to further this end." It is impossible here to give the details of the program of the German subcommittee of the I. M. U. K.; it represents moreover the needs of a private organization. This program has greatly influenced the recent official syllabi, however, and will continue to do so.

In 1924 the Prussian Ministry of Education published a memoir entitled "The Reorganization of Secondary Education in Prussia." This served as forerunner of the changes of the following year. It states in general educational terms the principles underlying the proposed reorganization and closes with programs of study for the various schools. Some idea of its contents can be gained from two quotations from it. The first traces the historical development of the reform movement; the second shows how mathematics and the sciences may have a genuinely cultural significance for students in technical schools. The first—"In the nineteenth century, and especially during the reform movements of 1890 and 1901, the educational ideas which recently have come into such favor were being advocated by school men and school boards of discernment. These ideas failed of complete acceptance at that time, however, because the reformers were unable to overcome the force of current opinion. . . . The chief effort of these educational seers was directed toward the ideal of a general well-rounded basic education suitable for all. . . . The cultural tenor of those days, the dominance

of Hegel's philosophy as to what Germany and the Germans should live for, and the new humanistic attitude toward civilization, militated against the acceptance of this ideal. But the overthrow of the Hegelian philosophy, the advent of the new realism, the differentiation of cultural interests, the increased specialization of knowledge, the immense widening of the scientific horizon, and the altered political situation of Germany have given rise to a totally different conception of general education.'' The second quotation goes on to describe how the schools through conscious education for German citizenship must play their part in meeting this changed philosophy of the state, and the assistance they have a right to expect from mathematics and the sciences in realizing this program. ''Teachers of mathematics and the sciences must not confine themselves to a narrow technical treatment of their subjects, but must lay greater stress than heretofore on their cultural, philosophical, and spiritual aspects as essential to the formation of the modern spirit and the new culture. In the past the danger of isolating these subjects, with consequent hurt to well-rounded instruction, has not always been avoided. Teachers of German, geography, and history have not always comprehended the educational import of mathematical and scientific studies and have even adopted a defensive attitude toward them, confusing realism with crass materialism and a mechanistic conception of life. The root of our national philosophy, so grievously in error in recent decades, lies really in a scientific dilettantism which can be combatted only by making instruction in science more thorough and deep, more truly scientific.''

''The Objectives for the Courses of Study in the Secondary Schools of Prussia'' appeared in 1925. They are in accord with the aims expressed in the foregoing quotations and endeavor to point the way to their attainment. For example: ''Problems of instruction should be considered not in terms of a certain subject but in terms of the accepted principles and aims of educational effort; and opportunity must be afforded freely for independent and creative work in the classroom. This is in obvious contrast to the severely systematic courses of study of former times and allows latitude for the untrammeled development of individual abilities.''

The educational significance of the connection between mathe-

matics and the exact sciences appears best in this statement of aims for the technical schools. "Intensive study of mathematics and the sciences accustoms the youthful mind to clarity, reality, the inevitableness of law, and the desire for truth; it gives familiarity with methods of thinking which are applicable to all mathematical and scientific work (*e.g.*, functional thinking and learning by induction); and the close relation between mathematics, the sciences, and philosophy tends increasingly to turn the attention from the particular to the general, and to develop the power to formulate general questions succinctly and clearly. To trace again the steps by which the mind of man has arrived at the results, methods, and problems familiar to us to-day gives a wholesome respect for the creative mind and a real appreciation of what genuine intellectual effort involves. Acquaintance with the modes of thought of other people, their mathematical and scientific output, and their application of these to industry makes the student aware of the essential unity of spirit embracing the whole world; while on the other hand a knowledge of the special scientific achievement of his own country and the resulting social problems gives him a nationalmindedness which is highly desirable."

Turning now to the teaching of mathematics itself, the objectives are announced as follows: "Accuracy and speed in the fundamental operations of arithmetic, with applications to problems of citizenship; appreciation of quantitative relations; practice and mastery in mathematical operations which lead to the realization that mathematics is ordered and extensible; ability to see the mathematics involved in the shape, measure, number, and law which governs the objects and occurrences of our daily life; ability to make independent application of these; development of space perception and ability to see how one quantity varies with and depends upon another; skill in logical deduction and proof; and a sure grasp of the philosophical implications and cultural significance of mathematics."

These objectives are to be attained by "constant adaptation of the mathematical material to the ability of the pupils . . . in accordance with the latest educational doctrine. Only such subject matter and skills shall be retained as have general educational significance or the possibility of practical application. . . . The work should not be confined to exercises and (old style)

problems only, but should include the investigation and discovery of new (*i.e.*, to the student) propositions in geometry and algebra, and to skill in drawing inferences and constructing proofs. . . . Applied problems should be taken from actual situations in the outside world and their solutions should lead to practical results of some significance. Correlation with other school subjects should be made wherever possible, and especially to economics and business problems." Special value is attached to the history of mathematics not only for class discussion but for pertinent mention in the assigned problems. "The part played by mathematics in the cultural development of the race should not be neglected; and students in the upper grades of the secondary schools should become acquainted with mathematical writings of historic significance."

Algebra must grow out of the arithmetic of the elementary grades. "It begins with the tabulation of formulas by substituting numerical values. The fundamental laws of algebra are to be treated as generalizations of the familiar operations of arithmetic, and exercise in their use should be tied up so far as possible with equations of the first degree. The same holds for the treatment of proportion. The function concept should form the core of the instruction. It should be introduced intuitively at first, be brought gradually to the fore, and receive a broad general treatment in the upper classes. Concomitant with this should be the development of the concepts of continuity, slope, and area. . The notions 'unknown and equation' and 'variable and function' should be clearly distinguished. Through the differential and integral calculus the student gains familiarity with the most powerful of mathematical tools. At all times a nice balance must be maintained between practical needs and mathematical rigor."

"In geometry an early consideration of the simple solids is desirable for the development of spatial imagination and intuition, and for the accurate representation of three-dimensional figures by means of drawings; this work also affords practice in estimating, in measuring, in the use of ruler and compasses, and in the construction of models. Instruction in geometry should afford a view of the systematic structure of the subject, and in proceeding from an empirical beginning to a greater and greater insistence on logical deduction should make the student

realize the need for proofs and lead him from mere knowledge of geometric facts to a real understanding of the subject as a whole. Motion should be used freely from the very beginning, and it should become a habit to consider the effect which the variation of one part of a figure has on the size of the remaining parts. Special emphasis should be laid on axial and central symmetry, and on the translation, rotation, and overturning of geometric figures.'' Exercises in drawing and measuring are especially valuable; the work in drawing should be developed from orthogonal (parallel) projections on a plane as is done in the text on descriptive geometry by Scheffers-Kramer. It remains for actual trial in the classroom to show how far this is practicable. The usual practice hitherto has been to begin with projections in plan and elevation. The work should include the making of models out of wood, wire, paper, and thread; and colors and other devices should be used freely to indicate geometric properties. From the writer's own experience over a period of years, students in school can achieve astounding results in the construction of models; and pupils in the intermediate grades of the secondary schools have even been known to execute quite difficult models in glass, such as inscribed and circumscribed cubes, octahedra, dodecahedra, and icosahedra. The representation of slopes of streets, of road intersections in hilly country, and of all sorts of roofs arouse great interest and assume a great variety of forms under the students' hands. Very recently there has been an effort to introduce nomography (*i.e.*, graphical computation), either in connection with analytic geometry or with algebra. There has been extended use also of graph paper printed with logarithmic and other scales.

Practice in geometric measurement, which in the lower grades is confined to simple measurements of heights and distances on the ground, is extended in the upper grades to include independent work by groups of students on mathematical projects. Besides measuring the areas of plots of land by means of polygons, and running street lines (preferably in a closed traverse), the students do some differential leveling. This sort of activity reacts favorably on their written work, as evidenced by their comments on the underlying theory and by the carefully tabulated observations, calculations, maps, plans, and profile sketches which come to the teacher's desk. It takes but a few years

to obtain in this way a complete map of the environs of the school. The accuracy demanded in this measuring and surveying should be gradually increased so that the students may come to appreciate the tremendous difficulties which geodetic and astronomical workers have to meet.

Since one aim of mathematical instruction should be to give a clearer understanding of the import and significance of mathematics itself, there should be a systematic review of elementary mathematics with this in mind. This would bring to light the inner meaning of the whole mathematical structure, and its influence upon philosophy and our conception of the universe. "Teachers of mathematics should not be content with giving their pupils a thorough knowledge of mathematical processes, but should arouse their interest in the systematic structure of the subject as a whole. Facts from the history of mathematics which are milestones in its development, and especially interesting bits of biography, can be of service in this regard. The history of the growth of mathematics (and the sciences) is a part of the history of the cultural development of the human race. It is recommended that the problem material have so far as possible a historical flavor; the classical schools will find their ancient history very helpful in this regard. Occasionally it will prove instructive to trace the historical development of some particular subject or topic. With all this, teachers of mathematics should bear in mind that their task is only a part of the larger task of training for citizenship. Problems drawn from the community, from politics, from industry and commerce afford frequent opportunity to connect the instruction with life. Questions affecting the national economy, food supply, the production and consumption of goods of all sorts, and industrial statistics, offer material for graphic representation in the intermediate grades; and problems in exchange and insurance are suitable for the last years of the secondary school." The intimate relation between mathematics and science should be mentioned here, and the teaching should indicate that "scientific precision, ordered by mathematics, is guardian of the laws of nature." Greater use should be made of geography, map making, surveying, and astronomy, accommodating the method to the nature of the class. Since mathematics deals so largely with drawing, it is but natural that the instruction in geometry

should be related to art and architecture, and should show from suitable examples how sketching and painting are based on the laws of perspective. And finally, instruction in mathematics should not hesitate to become speculative and philosophical. Questions of logic and the limits of knowledge are well within the purview of mathematics, the more because mathematical concepts have been increasingly employed by philosophers since the time of Descartes and Kant. Psychological considerations such as the meaning of number and space perception take on added significance when treated in conjunction with mathematics.

To give in detail the content for each grade would lead us beyond the limits of this paper. I shall confine myself to a statement of the chief differences between present day instruction and the immediate past. The increased emphasis on geometric drawing has already been mentioned. In the seventh and eighth grades an intuitive treatment of geometric solids makes possible a parallel treatment of the geometry of two and three dimensions, beginning in the ninth grade. How this is to be consummated is left to the individual teachers and schools, though the official statement of objectives gives definite examples as to how such a treatment can be developed.

Proportion is no longer regarded as a separate topic, but is taught incidentally in connection with the function $y = mx$ as a special case of the linear function.

Logarithmic calculations are now confined to four significant figures. The theory of the slide rule and skill in its use are taught almost universally.

Permutations and combinations are no longer taught except in the technical schools, and there only as brief introduction to the subject of probability. Algebraic geometry and so-called modern geometry have been stricken from the course of study. The latter included the theorems of Menelaus, Ceva, Pascal, and Brianchon, and the Euclidean treatment of polars; what little is retained is taught in connection with conic sections.

This omitted material has been supplanted by an elementary treatment of the theory of functions of a complex variable, an innovation which has been the subject of much controversy; it still remains to be seen whether this bit of mathematics of university grade can endure. There are several places where the linear functions of a complex variable might conceivably be

introduced to advantage; for example, in connection with the study of transformations in analytic geometry. But the treatment of even the simplest non-linear function, $w = z^2$, presents great difficulties in that it involves not only the question of double valuedness but also a two-sheeted z-plane.

The geometry of the last years should center around the conic sections. In accordance with the syllabi of the German subcommittee of the I. M. U. K., the treatment should be a continuation of elementary geometry, beginning with the Dandelin spheres and ending with a presentation of the method of Apollonius, and contrasted with the treatment of the conics in analytic geometry and as central projections of the circle.

These in the large are the objectives laid down for the Prussian schools. We have confined ourselves to Prussia because these objectives have been accepted almost universally throughout Germany. Even where divergences do appear, they do not affect the essential spirit of the objectives outlined above.

And now a word as to the attitude of the teaching profession toward these changes. The idea of fusing plane and solid geometry and the introduction of measuring and drawing are still hotly contested. The simultaneous treatment of the geometry of two and three dimensions, which Euclid took such pains to keep separate, was first undertaken in Italy under the leadership of Enriques. Experiences there, however, have created an aversion to a systematic fusion of these two branches in Germany, where opinion favors a moderate fusion, as indicated in the official "Objectives." In connection with the computation of areas of two-dimensional figures, for example, the volumes and surface areas of solids are computed, and drawings are made of roofs, houses, and geographic formations.

How far the idea of a transformation—and a group of transformations—should be carried, is not yet clear. I have already mentioned the widespread objection occasioned by the proposal to introduce complex functions. In my own text I have treated the transformations all together, being guided by the results of personal experiment in the classroom. Whether this has been done elsewhere or not, I do not know. I should suppose that the work with complex transformations should not proceed beyond the general linear complex function $w = az + b$, where the variables and constants are both complex; although possibly

the best classes could do something with $w=1/z$ and $w=z^2$, considered as transformations of the plane.

There is almost complete unanimity regarding the treatment of the conic sections, as previously described.

The introduction and extension of the number concept is commonly based on the idea of Permanence of Form. Whether one should go further and define the irrational in terms of classes of numbers and the Dedekind Cut, is an open question. The latest textbooks indicate a tendency in that direction.

Teachers of mathematics are agreed that the work in algebra should center around the function concept, and the official "Objectives" support them in this. But whether the work should begin with empirical formulas, or not, is still the subject of much contention. A large number of meritorious textbooks represent efforts in this direction. Some even go so far as to recommend for the highest grades the treatment of formulas drawn from statistics and the theory of errors, including the Galton curve, the Gaussian error curve, and the notion of correlation.

Equations of the first, second, and third degree—formerly a favorite source of mathematical exercises—have, naturally enough, been crowded into the background by this movement. There is unanimous agreement, however, that the work in equations should come much earlier than heretofore and serve to introduce the idea of function. The former emphasis on graphic and approximate methods of solving equations in the last years of the secondary school has given way to a purely algebraic treatment intimately bound up with the function concept.

A lively discussion is going on at present concerning the method of presenting the derivative, some favoring a wholly intuitive and geometric treatment which others just as earnestly denounce.

Despite the fact that the official "Objectives" give great emphasis to geometric drawing and measurement, there is no insistence on a thorough course in descriptive geometry such as is found in Austria. This is frequently deplored, and it is earnestly hoped that provision may be made in the upper grades for at least a year of descriptive geometry.

It is impossible to state in detail the agreements and divergences between mathematical publications and the ideas herein expressed. Without in any way attempting to give a complete

list, I give here the names of four texts for school use which reflect the spirit of the reform movement:

1. Lietzmann-Zühlke. "Mathematisches Unterrichtswerk." 4 books, with few explanations, but rich in exercises.
2. Lötzbeyer-Schmiedeberg. "Mathematik für höhere Schulen." 5 books, exercises and text combined.
3. Malsch-Maey-Schwerdt. "Zahl und Raum." 8 books, exercises and text combined.
4. Reidt-Wolff-Kerst. "Elemente der Mathematik." 4 books, exercises and text combined.

The third text, of which the writer of this article is joint author, is reviewed in this number of the MATHEMATICS TEACHER. Supplementing the regular texts are a host of ancillary publications intended for school use. Chief of these is "The Library of Mathematics and Physics," edited by Lietzmann. This collection comprises about 62 monographs of from 40 to 80 pages on a great variety of mathematical subjects. Some of the titles are: Number and the Number System; The Mathematics of Finance; Conformal Mapping; Nomography; The Slide Rule; On the History of Mathematics; The Squaring of the Circle; The Gyroscope; and so forth. Then there is "The Mathematical Reader" by Dieck and some historical works by Wieleitner.

On the professional preparation of teachers there is an instructive book by G. Wolff entitled "Zur Frage der Ausbildung der Lehrer der Mathematik und der Naturwissenschaften." This is one of a series of articles appearing as supplement to "Die Unterrichtsblätter für Mathematik und Naturwissenschaften." This journal and its supplements (Beihefte) contain many worthwhile contributions; the same is true of "Die Zeitschrift für den mathematischen und naturwissenschaftlichen Unterricht" and its supplement.

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SOME SUGGESTIONS ON THE TECHNIQUE OF TEACHING PLANE GEOMETRY

By E. B. COWLEY

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Probably every experienced teacher who reads this article has his own method of teaching plane geometry, which he has been gradually developing for many years. In his own hands his method is something live, keen, and powerful. But if another were to attempt to take that method and use it just as it is, he would find it a dead and uselesss thing. If anyone wants to make profitable use of the methods of another, he must transform them into harmonious accord with his own personality and the needs of his class. On the other hand, it may be quite worth while for us to hear other teachers discuss their methods; for, although we may have no desire to make any direct use of the plans set forth, a chance phrase may suggest to our minds a new line of attack upon some weak point or some obstinate difficulty in our own methods. It is with this thought in mind that I have written out this account, although I claim no particular merit for my methods.

The methods which I have been using this year in high school classes in plane geometry are adaptations of a general scheme which I have been developing for the past six years. I began working along this line with a college class in descriptive geometry and mechanical drawing where my students ranged from seniors who were honor students majoring in mathematics to sophomores who had had nothing beyond solid geometry and plane trigonometry. The method was further developed in college courses in elementary and advanced analytic geometry and in calculus. This year I have had my first experience in trying it on classes (eight of them) in plane geometry.

The first few weeks of the first semester's work were, of course, given to preliminary work when the pupils were becoming acquainted with the new terms and learning to make the constructions that would be needed in Book I. As we progressed into the proofs, my assignment consisted of a small group of closely

related theorems. At the beginning of a period I announced an assignment, giving such explanation as was required in each particular case. My aim was to arouse the pupil's interest and curiosity, to suggest the relation of the new work to previous work, and to encourage the pupils to compare the new theorems with one another and with theorems they had already studied, noting points of similarity or of difference. There is great danger of making the explanations so full and complete that the pupil depends upon his teacher to do all his thinking for him. Under such conditions he does not gain self-reliance, nor does he experience the thrill that comes only with the real mastery of a difficulty.

After the assignment had been explained, each pupil set to work for himself in his own way—provided that he really worked and that his activities did not interfere in any way with the peace and happiness of anyone else in the room. Some practiced at the blackboard; others preferred to work at their seats. Frequently two pupils decided to work together for a while. Anyone who encountered a difficulty which he could not handle alone was free to seek guidance from me or from any pupil. Meanwhile I was making it my business to know what each pupil was doing every day and how he worked. I was on the lookout for the superficial and the slow and idle, as well as for the gifted boys and girls.

As soon as a boy thought that he had mastered an assignment, I gave him an oral test. Usually he drew his own figures on the board. But if I found that a pupil was memorizing without understanding, I either relettered his figure or drew a figure for him myself. If he failed, he returned to his seat for further study and tried another test several days later. But if he had really mastered the assignment, he was given another unit of work. Sometimes this was a set of exercises which would not be given to all the class. Sometimes I indicated the theorems which would constitute the next assignment to the whole class and let him attack it without any explanation. He started out as a courageous and enthusiastic pioneer to explore this new field alone.

When the majority of the class had mastered an assignment, it was announced at the beginning of a period that a written lesson would be given two days later. Everyone in the class

was required to take this written test, no matter where he was in his work. In these two periods a fine spirit of cooperation was exhibited. Everyone wanted the class to do well on the test. Frequently several of the ablest pupils who were ahead with their work were requested to assist those who were behind. It was considered an honor to be invited to act as a tutor, and a privilege to receive assistance. The comments which these pupil tutors made upon their fellow pupils were surprisingly discerning and were frequently brutally frank.

Great care was exercised in the selection of suitable questions for the written lessons. From my experience in marking answers to examinations which I have made myself and those set by others, both as a reader on the College Entrance Examination Board and in college work, I have come to the conclusion that, while a failure may be due to the candidate's ignorance or dullness, it may also be due to the stupidity of the person who made out the questions. Occasionally I took two consecutive days for written tests—one for proofs of theorems and the other for simple applications. My written work was returned the next day. I spent a few minutes explaining my marking system and any other points that were of interest to the whole class. Points that affected only a few were taken up with those individuals later.

To avoid the monotony that would result from too close an adherence to this method, and to attain other ends which will be mentioned later, various innovations were introduced at irregular intervals. Sometimes an assignment consisted of exercises, with some required and others optional. Occasionally a boy who had worked an optional exercise presented it to the class. One day a big boy who preferred reading scientific magazines or drawing cartoons to studying, gave a really creditable presentation of an article describing a detail of construction of an aeroplane. Once or twice we spent a day on geometric fallacies or on construction puzzles. Once I told them of a simple device for getting the heights of trees. Again I told them how we had used the Pythagorean theorem to run the boundary of a lot in a Maine wood-pasture. Occasionally we devoted an hour to what our colleagues in the classical department would call "sight work." I selected a new theorem whose proof involved no unfamiliar theorems and had them open their

texts at that page and gave them, say, five minutes to study. This gave me an excellent opportunity to observe their powers of attention. It gave them a realization of what may be done in a short time. Books were closed promptly and explanations called for. Occasionally I dictated a new exercise and gave a limited time for solution, allowing them to consult their books freely (but not one another).

In developing this method of work there are certain aims which I have been trying to realize:

1. To recognize the individual differences of ability and temperament in pupils, by allowing them to work at that speed and by that method best suited to themselves. The able pupil is stimulated to do work worthy of his powers and at the same time the slow or dull boy can gain self-confidence by attending to his own business, instead of being discouraged by constant unfavorable comparisons.

2. To give every pupil an opportunity for self-expression—having seen to it that he has something to express. In the ordinary recitation, the able pupil monopolizes the class time, except on those occasions on which, from a sense of duty, the teacher bores himself and his class by trying to force the slow or dull boy to recite. But still worse is the "lecture" system where the teacher is really an expert entertainer, talking hour after hour to an audience which may be sleepy and passive, or mischievous and restless.

3. To provide opportunities for the development of initiative and originality.

4. To impress upon every pupil the fact that he is personally responsible for mastering the course.

5. To lead the pupil to compare theorems, to hunt for likenesses and differences, to rearrange and reorganize his facts, and thus learn to reason; avoiding the mere rote memorizing that is sometimes mistaken for study of geometry.

6. To save the pupil's time and to teach him the value of time. I am appalled at the waste of time in schools and colleges. In the public schools we pride ourselves upon our thrift work, but could not the pupils be led to see that time is as valuable as money? Is it not frightfully extravagant to allow the loss of time that occurs in the ordinary recitation, or still worse in the lecture?

7. To bring the work of the class to a focus at intervals by written tests. By this means I have tried to gain some of the class spirit of cooperation that seems to be lacking in some methods which stress individual work.

8. To provide sufficient variety to avoid monotony, and at the same time to furnish enough uniformity to secure continuity.

9. To give the teacher more time to become acquainted with the pupils' minds and thus to be able to render more intelligent guidance to them. The individual oral and blackboard test not only brings the teacher face to face with an individual pupil, but also eliminates some of the burden of correction of written papers which can destroy the vitality and enthusiasm of the teacher. Moreover, the pupil who receives too many written tests is apt to grow indifferent to the comments and corrections made on them by the teacher. There are so many other more profitable ways of spending the teacher's time and energy. The teacher needs plenty of vitality, for every new class is a challenge to the real teacher. As you look into the face of each new pupil, you ask yourself, Why is he in school? Is it the compulsory school law, or is he eager to learn? If he has a definite purpose, can I do my part and see to it that he does his part in achieving this purpose? If he has no definite purpose, how can I arouse him? Will he learn geometry in my class in such a way that he will incidentally learn to think logically, to look at both sides of a question, to be honest and industrious in his habits?

These are some of the ideals toward which I am working.

LEST WE FORGET [1]

By THEODORE LINDQUIST

State Teachers College, Ypsilanti, Mich.

In these days of project-problems, attempts at fascinating appeals to the pupils of our schools, as well as the rapid transit in the topics taught in our classes, may it not be worth our while to stop, look, and listen to find if we are not in the danger zone of forgetting some of the fine old golden principles that have stood the test of time. There will be no attempt to decry the good new things but to recall some of the good tried things lest these be forgot. Nor is even this idea of recalling good old standards new. The fine old gardens of our grandmothers are coming into their own again with the good old-fashioned "pineys," phlox, snapdragons, mignonette, and so on.

The writer was shocked beyond measure a short time ago to hear the superintendent of schools of a large western city say, that in the organization of their junior high school they were to discard all the work that had previously been given in the seventh, eighth, and ninth, the junior high school grades. Think of the fine old material tried out through the decades, not to say centuries, that he would throw away just because it had been used for a long time. Really, what is our education coming to under such advocates. It is far from the intention of this paper to advocate either the retention of the old just because it is old nor to hold out against acceptance of the new just because it is new. The cry is against acceptance of the new as a panacea just because it is new and the throwing away of the old just because it is old. Just one illustration: Some thirty or forty years ago there was a movement to replace the sexagesimal system of angular measurement with the decimal system. How many to-day have even heard of this movement? It died a natural death as such impractical fads do. We still use the good old system devised centuries ago.

One of the things that we are prone to forget in the teaching

[1] Delivered before the Mathematics Conference of the Michigan Schoolmasters' Club, Ann Arbor, Mich.

of mathematics is what our main objective should be; namely, that every pupil makes the most progress in his mathematics that he is capable of making. With our large classes of the present it is difficult not to neglect the brighter pupils in our efforts to being the mediocre students up to a passing grade. In the larger school systems where the pupils are divided into groups according to their ability much can be done in this matter of securing the very best efforts from each student. Even then the teacher must be continually on the alert with the pupils of the brighter section or valuable time will be lost. A superintendent once told the writer that the pupils of his high school had been divided into sections according to their ability. As he was a mathematics teacher himself he taught the section of poorer pupils. The section of brighter pupils he delegated to a teacher who was not primarily a mathematics teacher. The sections of average ability he gave to the regular mathematics teachers. The results at the end of the year were a sore disappointment to him. Each section had covered practically the same amount of ground both laterally and vertically. This is mentioned to point out the fact that the arrangement did not in itself produce good results. What we need more than special arrangements is good mathematics teaching by teachers prepared to teach mathematics. The general sales manager of a branch of a much used commodity, who has charge of a large western territory, told the writer that the main requirements he makes of his salesmen is that they know thoroughly the product that they go out to sell. His slogan with his men is "K. Y. P."; know your product. Should we not add here for the mathematics teachers, K. Y. M.; know your mathematics. Know what it is; know its value and how it can be applied; know how to teach it efficiently to others. In addition to this, real efficient mathematics teaching demands that the teacher also must possess a professional interest in the subject matter as well as a humanitarian interest in the pupils and in this nation.

The teacher who has classes composed of pupils with varied abilities in each class has a harder problem but it is not an insurmountable one. The minimum essential lesson has been tried with good results. That is, a minimum amount of work is assigned to all the pupils of the class for the next day's lesson. The brighter pupils are then encouraged to go into the subject

more thoroughly. One method of accomplishing this is by the solution of more difficult problems by these brighter pupils. Where the class period is sufficiently long to provide for supervised study, this method can, of course, be carried out better than where there is no such provision of time. Even under the favorable conditions of the supervised study arrangement good teaching is as necessary as it ever has been before. With the careless teacher who is not imbued with a professional interest in the work or alive to the needs of the pupils, it becomes merely an opportunity for the pupils to get work out of the teacher without doing much themselves.

Recalling our proposition we note that the *pupils* are to make progress in mathematics. In the example cited where the pupils had been divided into three groups according to their ability the pupils in the slowest moving group evidently made the most progress. Their teacher secured the most work from his pupils while the teacher of the brightest group secured the least amount of work from his pupils. Teaching as the latter is not professional and it is far from fair to the brighter pupils. Success in class teaching can not mean bringing all the pupils of the school or class to the same level. It must mean the elevation of each pupil to the very greatest height to which he is capable of rising.

Daily we hear and read so much about interesting our pupils in their work. All good teachers with a professional interest in their work are in full accord with this movement. Securing the interest of the pupils of a class is the least part of the total scheme, however, of working up interest. The main object should be to secure work from the pupils; the matter of securing interest is *only* a part of the means to the end. Yet in how many cases where we hear discussions on securing interest, little or no mention is made of the fact that securing the interest is just to be an appetizer for the real work that is to follow. Far be it from any of us to advocate making the work in mathematics uninteresting. Let it be made just as interesting as possible but our main objective as teachers should be to have our pupils make progress in mathematics and not be interested in some problem or device. Children should eat to get nourishment, to grow, and to become healthy men and women. By all means make the food as tasteful as possible, providing that it

still furnishes the nourishment necessary. The writer knew a teacher who by using a play store and a play bank secured the very best results in mathematics work of her pupils. Later on she became so imbued with the idea of securing interest and of working out projects that these became the main objectives of her teaching. The whole matter which had been such a fine source of securing mathematics progress became little more than mere play. She had changed her objective from making progress in the study of mathematics to that of evolving new projects. There is danger in project-problems becoming so much project that sight is lost of the real essential, the problem.

It is necessary for the teacher to distinguish between genuine interest and fleeting fancy. In the former case the pupils will work hard and will be proud to secure results; they will make progress in their studies. In the second case they will soon tire of what they are doing and will desire to flit to something else. Here again it takes a teacher with ability and professional insight to distinguish between the real and the fad. It is the real good old-fashioned work that counts.

Let us here recall an old principle which is absolutely essential to good teaching of mathematics. Make progress slowly. Take up only one new principle or operation at a time and be sure that it is thoroughly mastered before another is attempted. Progress in mathematics is similar to mountain climbing; it is not the height to which one goes that makes the going difficult but the grade that is used in going up. If the ascent is only gradual enough, great heights can be attained easily. To carry the simile a little further, suppose that a group of mountain climbers proceeds up a mountain side by a winding path while a few laggards loiter behind. After a while the laggards see the rest of the party on the path directly above them. Their mad scramble to go directly up the steep side to join the main party is not only most difficult but often impossible. How many mathematics teachers have not witnessed this very thing in their classes! One principle and one operation at a time with sufficient time spent on it to make a lasting impression. No, we do not believe in the rigid topical process of studying one principle or group of principles for a comparatively long time and then proceed to seal them up in a jar to be placed upon a shelf to gather dust. Far from it, but each principle and opera-

tion should be given sufficient attention to make a lasting impression on the pupils. A mad flitting from one thing to another will produce little progress in mathematics at the end of the year and less by the beginning of the next year.

A second principle that is fully as necessary in securing good results in the teaching of mathematics as that just mentioned is that of constant reviews. Especially is this true for the earlier work; say, through the high school. When should reviews be given? All of the time. Review, review, review, and then keep on reviewing. At the beginning of the year's work it may be well to review particularly those principles and operations previously studied which will come into play most frequently during the progress of the year. In order to make the greatest progress possible during the year this review should be elastic so that it can be emphasized with the class needing the most review and lessened with the class needing less review. We all realize, to be sure, that a formal review of the previous year's work at the beginning of the school year will be more or less fatal. The pupils feel that they are making no advance. Reviews, to say the least, are like left-overs in the kitchen. We do not like the same meal to-morrow that we enjoyed to-day. But these left-overs can be made into appetizing dishes by the skillful cook. The same can be done with reviews in mathematics. Professor Bagley has aptly said that we should give "new views" rather than reviews. One illustration of a somewhat elementary nature must suffice. Checking computations by casting out the nines may well be introduced into the junior high school work as something new when work is begun in the fall of the seventh grade. In order to apply this check to multiplication and division the pupil must first multiply and divide; that is, review multiplication and division.

The most effective review, however, is the daily review which is wholly disguised. In this form of review problem material is so selected that any principle or operation previously studied arises from time to time in the successive lessons. This form of fusion of mathematics will never lead to confusion. The operation studied to-day will be made real to the pupils by its later application to various kinds of problems. For instance, after our pupils have learned the square of a binomial, as $[a+b]^2$, we should see to it that they repeatedly have use for this later.

We should also insist that this short-hand process be applied whenever opportunity offers itself as in squaring 254 which they could square as $250 + 4$. Another reason why this is the most effective review is the fact that here the pupils meet with a real situation in the matter of applications of what they have learned previously. In fact, this is as nearly the situation they will meet outside of school as can possibly be provided for them. Furthermore, in this way they come to know that they will later find use for the material in the lessons of to-day.

Summing up we would say that the main objective of the teacher of mathematics should be to get the pupils to make progress in mathematics; to learn to think mathematically. The real true meaning of this is that each pupil should make just as much progress as it is possible for him to make with his capacity. Reviews are necessary at all times and they can also be used to cement the work together into a unified whole without creating any confusion in the minds of the pupils. It is further necessary to make progress slowly, to go from one principle to another only after the one studied to-day is understood clearly and can be applied successfully to the solution of problems. No artifice can be made to take the place of real honest hard work on the part of the pupil in accomplishing that result. The teacher will be well paid for interesting the pupils in the tasks before them but it is the work that the pupils put forth because of this interest that really counts. The successful teacher must know his product. He must be thoroughly conversant with the subject matter and imbued with a professional interest in teaching the same while he has a real humanitarian interest in the progress of his pupils and in the welfare of the nation.

AN ATTEMPT TO IMPROVE COMPUTATION

By MARY POTTER

Supervisor of Mathematics, Racine Public Schools

"The bananas are 30c a dozen, 4 for a dime, or 8 for 20c," the clerk at the little store at the summer resort informed me.

"All right, I'll take 9," I ordered innocently.

"Oh, I can't sell you 9; I don't know how much that would be. You can buy 8 or 12," she replied.

In vain I insisted that I taught arithmetic, and in my most painstaking classroom manner explained the price of 9 bananas at 30c a dozen; she was unconvinced even at the end of 5 minutes, so I trudged back to camp with *eight* bananas under my arm. For some time thereafter my pupils were carefully instructed in buying and selling articles by the dozen.

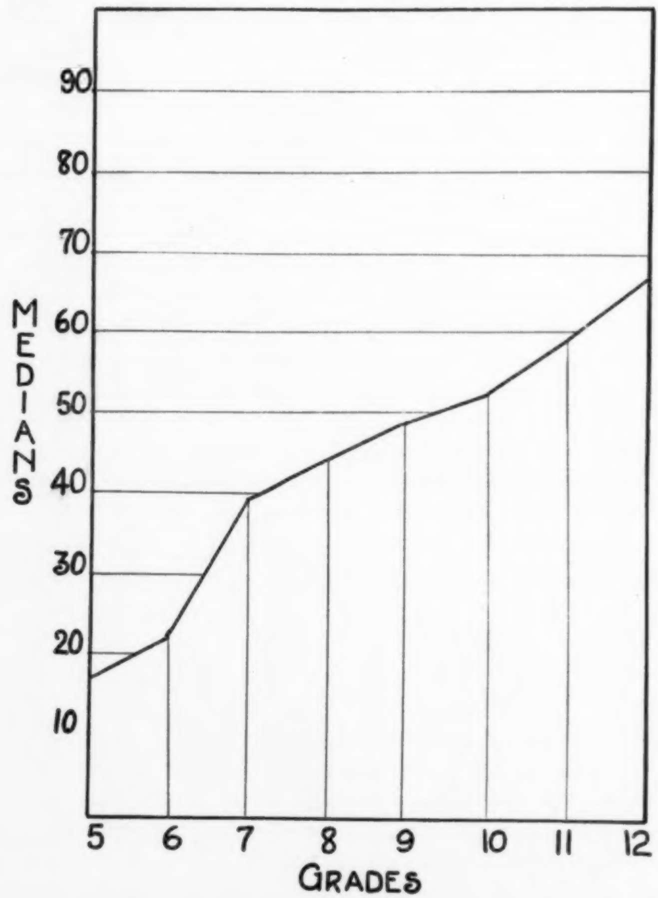
For three months in succession my telephone bill came wrong. The annoyance of the necessary repeated adjustments resulted in a series of lessons on printed bill forms with great attention paid to correct addition of short columns of figures.

Then Christmas time rolled around with its usual accompaniment of church fairs and cafeteria suppers which we found well worth our patronage. At the first affair I was short changed, but one hesitates to mention being short changed at a church function. At the next supper I felt compelled to hand back the extra change I received. After that I played a new kind of solitaire, accepting silently whatever change they gave me, but keeping score. In the end I came out about even, but I received the correct change only once. These small business transactions were all made with women who received their sole formal business training in our schools.

But why repeat your own experiences and accumulate evidence to prove the contention of the all-too-tired business man that we don't annoy ourselves to bother to teach our pupils arithmetic? Educational studies as well as our own observations show that our eighth grade graduates do not compute with even a reasonable degree of accuracy.

Feeling that the taxpayer is entitled to a higher degree of

arithmetic skill in the product we are turning out and convinced that the quality of our advanced mathematics would be greatly improved by a mastery of the processes in the basic mathematics—arithmetic—we centered our attention upon computation.



From tests, both standardized and informal, and from careful observation of errors in daily lessons we learned many unpleasant but illuminating facts. But wishing the more definite information that we could get only from a more elaborate survey, we gave a test of 100 simple tasks [1] involving the ordinary

[1] The Schorling-Clark-Potter Survey Test Form *A*.

arithmetic tricks of addition, subtraction, multiplication, and division of whole numbers, common fractions with the easiest denominators, decimals, percents, denominate numbers. Each process was sampled and care was taken to shift attention as little as possible by such means as grouping the 10 addition exercises together. We found not only that whole numbers were not handled well, but fractions were abused; no respect was shown for the decimal point which by its very position rules a number more absolutely than ever a czar ruled his millions; denominate numbers and percentage were passing acquaintances.

To be more specific, no task was handled perfectly by any grade; the best score of 98 percent was made by the twelfth grade in subtracting 742 from 1,124. Contrary to our expectations the twelfth grade made the highest median, a score of 67 out of a possible 100; from the twelfth grade the medians gradually decreased grade by grade to a median of 17 in the fifth grade, as is shown in the accompanying graph.

Wide variations in skill were noticed in each grade, but the greatest variation occurred in the ninth grade, where the best computer did 94 tasks correctly in contrast with a score of 6 made by the worst computer. Evidently great skill in teaching is required to handle such a situation.

A glance at the table shows the familiar overlapping of scores in the different grades.

	Grade							
	V	VI	VII	VIII	IX	X	XI	XII
Highest Score	31	70	86	91	94	92	96	94
Lowest Score	1	3	6	9	6	17	16	22

Evidently the standard for promotion in computation is age rather than skill.

Only 9 out of 10 pupils could add a column of 7 figures whose sum was 46, and this was an almost constant performance from grades 5 to 12.

When told to find the average of 4, 6, 8 and 10, only 1 out of 10 fifth grade pupils was successful, and 2 out of 3 high school seniors.

Only a seventh of the eighth grade pupils and two thirds of the twelfth graders were able to arrange the decimals .40, 2.5 and .875 in the order of their size.

To many pupils a fraction is a fraction, so why be so fussy as to distinguish between the different kinds as common, decimal, or percent. If $\frac{1}{2}$ is to be changed to a decimal fraction, why isn't 50 percent satisfactory, or for that matter why bother about a decimal point? For such reasons only 56 percent of the seventh graders and 90 percent of the twelfth year pupils were able to change $\frac{1}{2}$ to a decimal fraction.

Denominate numbers proved to be greater mysteries than detective stories for only 9 percent of the eighth graders were able to find $\frac{1}{2}$ of 3 bu. 2 pk. 2 qt.

Doctors have long since been forced to recognize that diagnosis without remedy is fatal to the patient. Profiting by their experience we are trying to find out and apply the proper remedy for our ailing computation.

In the little old red school house days all children were drilled on the same set of problems at the same time until the majority of the class could "pass." This meant that the ablest pupils were learning little because they were kept drilling on facts they already knew, and to the slower pupils everything was vague because they were never given time enough to conquer their difficulties. Being human, both the slowest and the swiftest were bored.

But drill is the father of skill; courting skill we must study and obey the laws of drill which are found in the teachers' Blackstone, psychology. Chief among these are: Drill must be specific, it must be at least roughly standardized, it must provide a scoring technique, it must be individual, it must afford much practice on a few skills rather than a little practice on many, it must be diagnostic.

To obey the law that "a drill must be specific" we limited our drill [2] to specific subjects—whole numbers, and the more neglected fractions, both decimal and common, percentage and denominate numbers. The drills are arranged in groups, each group is devoted to one specific phase of a topic such as addition of common fractions, and each group consists of an inventory

[2] Instructional Tests (adjusted for pupils of varying abilities), a booklet for each grade from fifth to eighth inclusive.

test followed by a series of practice tests in which each practice test drills on some special unit skill in the addition of fractions. In an inventory test the pupil takes stock of his skill in that topic. If his score on the inventory test tells him that his stock in skill is low, he may acquire more by drilling on the practice tests which follow. Thus he may watch himself grow in skill in computing unhindered by the rest of the class; his drill work becomes a game in which he competes against his own score as well as the scores of his classmates, and he records his success upon a score card. In this way a class of 40 pupils may be working on 40 different lessons at the same time, each pupil repeating a lesson as many times as it may be necessary to master it, each pupil gaining some new skill rapidly or slowly, each pupil succeeding, each pupil enjoying his own success.

Experimenters have found that long drills are a waste of time and energy, that a short daily investment of time combined with the largest possible investment of energy yields the richest dividends. In accordance with this principle each drill is planned to occupy four minutes.

This is our program. We have our high ambitions that when our pupils have successfully completed their arithmetic drills they may be competent to fill out their own income tax statements.

LOOKING BACKWARD

By H. L. McCULLOUGH,

White Plains High School, White Plains, N. Y.

I read "Fads and Plane Geometry," by Mr. Merrell, in the January issue of your magazine with a feeling of admiration. Then I read it again and the glaring inconsistencies began to take form.

Mr. Merrell evidently thinks that parents ought not be interested in the schools. It is true that many of them are narrow-minded and selfish parents who seek privileges for their own children. A great number of parents do not realize that education is fast becoming a science and that teachers are becoming specialists in their subjects. The mother will take her boy to the doctor and accept his advice without a murmur, yet she will persist in exposing her boy to Latin or geometry in spite of the advice of the teacher who has diagnosed the situation. But there is another obstacle to educational progress—the teacher. We have become so accustomed to teaching as we were taught that it is well-nigh impossible for us to change our habits or open our minds to new ideas. And why should we? Have we not covered the ground, exhorted our pupils, detained them after school, passed eighty percent and failed the others? Besides, it requires much effort to test a new method and the chances are that it won't work. Then, too, we get excellent reports from the University about our star pupils who would have worked in spite of us, and we say to ourselves: "That's the joy of teaching. I am in the profession because I love children."

I am reminded of the story of a teacher who was unable to teach a third grade youngster the textbook material of that grade. Having tried various devices and failed, she classed him as a moron and had him sent to the city to a psychological clinic. The doctors put him "through the mill," found him above the average in intelligence, and sent him back to the teacher with their findings. When he appeared on the playground the next day, his friends taunted him with this remark: "Ah-ha, they

sent you away to be an idiot, and you took the test and didn't pass!"

I take it that Mr. Merrell hasn't much use for vocational guidance. I agree that it has been somewhat overdone, but what about those one thousand students who were turned down by the University? Have they no place in our educational system because they were not college calibre? He states that his brother, after a frank discussion with his father, decided to drop the academic subjects and become a farmer. Much time could have been saved if that particular school system had had a vocational guidance bureau, for the brother could then have been directed towards his life's work earlier in his school career. Perhaps he would have gone to the University for a course in scientific agriculture. But that is one of the new-fangled ideas.

Then the intelligence test comes under the hammer. I realize that many teachers look upon the I.Q. as final and decisive, but there are many more who use it as one of several factors in solving the pupils' problems. Mr. Merrell would have us refrain from using these tests until they have been established. Intelligence tests were given to thousands of soldiers during the war and have passed beyond the experimental stage. Even so, what better subjects can be found to experiment with in this way than our school children? I do not understand this remark about teachers not buying automobiles until they have been standardized.

He states that eighty-five percent of forty new students failed third-year algebra. When such a large group fail, either the students have low mentality or the teaching methods are at fault. Perhaps if the instructional methods were revamped and newer technique were introduced, the results might be different. Geometry and Latin do teach pupils to think, and think straight. When Mr. Merrell asked his students to reproduce a page in geometry, he was giving them a lesson in mechanics. What thought processes were involved in memorizing that page? Were those letter-perfect students more able to cope with the problems of life after memorizing that page? How often has Mr. Merrell used his information in geometry outside the class room? He spent ten minutes on the assignment and I imagine thirty-five minutes having the pupils recite what they already knew—a good argument for the introduction of new teaching methods. Try a ten-minute assignment of some foreign material

upon a group of teachers or other adults and see how many understand it.

He remarks that his former principal, Mr. Lane, was the first to introduce the so-called "suggestion method" of attack. I thought we school teachers should have nothing to do with these new-fangled ideas. No doubt Mr. Lane was criticized at that time because of this new approach in geometry. He did not wait until the method had been established but experimented upon his own pupils. It seems that Mr. Merrell is slightly inconsistent in this respect.

I heartily agree with the suggestion that teachers of geometry should be trained to teach that subject; but there is more than subject knowledge needed to teach any course. We need training in class-room technique, methods, ability to analyze the individual. We should be able to diagnose problem cases, to approximate the capacity of the student that he might be made to work up to his ability. This information can be secured from intelligence tests, achievement tests, psychological clinics, and other sources which are products of these "forward-looking experimenters." And these "cranks" are responsible for our normal schools, colleges of education, junior high schools, and junior colleges. Things are not as they used to be.

I think Mr. Merrell has a college complex. He thinks of the high school as a training-camp for the University. The majority of our high school graduates do not attend college, and if a course in geometry will help them become better citizens, then it is worth while. What difference does it make if the student does not retain all the information demanded by professors of mathematics in college? If the pupil has learned to think straight, to be critical-minded, and to evaluate results, the course in geometry has served its purpose. I, for one, am glad that the high schools are getting out of the grip of the college and are blazing the trail which leads to community and social adjustment. Of course, every student should go to college if he is financially able and has the mental ability, but I do not think he will be handicapped for life if he has no college education. The world is full of worthy people who have never seen the inside of a college or had the satisfaction of memorizing a page in geometry.

"FALLING IN LOVE WITH PLAIN GEOMETRY"

A Comedy in Two Acts

By CAROLINE HATTON and DORIS H. SMITH

Persons of the Play

Plain Geometry .... The Heroine
Anna Lytic Geometry .... Her Older Sister
Al Gebra .... A Cousin of the Geometry Girls
Phil Osophy }
Cal Q. Lus } .... College Students
Jim Nasium .... The Hero
Miss Terious .... The Disturbing Element
Major Arc .... A Friend of the Family

English
Latin
French
Home Economics
Physics
Chemistry
Trigonometry
Biology } .... Eight Girls; Loyal Subjects of the Curriculum

Act I: Living room in Plain Geometry's home; Saturday morning.

Act II: The same, Saturday evening, two weeks later.

Suggestions to the Characters

Plain Geometry—An attractive girl who could not in real life be called "plain" (in Act I); neat costume, suitable for a school girl. A simple afternoon or party dress in Act II.

Anna—A matter-of-fact older sister type. A house-dress in Act I; a suitable change for the party in Act II.

Al—A tall slender boy with a mischievous twinkle in his eye; has a keen sense of humor.

Phil }
Cal } Typical Collegiates.

Copies of this play may be obtained from Caroline Hatton, 537 West 121st St., New York City. Price 40 cents, prepaid.

Jim—A substantial, athletic, manly fellow with dignity and poise.

Miss Terious—A demure maiden of subtle charm; rather inclined to the clinging vine type.

Major Arc—A stout boy who is capable of military precision and command. For description of uniform see instruction for dance printed at end of play.

The Loyal Subjects of the Curriculum—Eight girls of same height and build. See end of play for description of uniforms.

Properties

Dust cloth for Anna.
Basket of flowers for Plain Geometry.
Ruler—on the library table.
Blackboard compass for Major Arc.
Eight yardsticks for the "Loyal Subjects."
A cent and a quarter for Al.
Small notebook and pencil for Cal.
Small drum and sticks for drummer in "The Advance of the Loyal Subjects."

Act I

Scene: Living room in Plain Geometry's home. The furniture should include a library table with a drawer in it; easy chairs, etc.; and a mirror on the side wall, either over a mantel, or in some convenient place. include a library table with a drawer in it; easy chairs, etc.; a mirror looking intently and discontentedly at her reflection.)

P. G.: I don't see why everybody calls me Plain Geometry. I get so sick of it. Plain! Plain! Plain! Everywhere I go I hear it. I wish I were pretty and attractive. (Stops and scans her reflection again anxiously.) I know Jim doesn't think I'm pretty. He likes that Miss Terious much better. Sly thing!—Even Jim calls me Plain. (Turns before the mirror, still looking at herself.) I wonder if it is because my figure is so angular, or because I am not acute enough. There *must* be a reason. (Pause.) There has to be a reason for everything..

(Enter Anna from left. Starts to dust chair industriously.)

Anna: What have you been doing all morning? Do you know it is nearly eleven o'clock? It's about time you were circling around and getting some work done.

P. G. (heaving a deep sigh): Oh! I suppose I might just as well get to work. That's all I'm good for, anyway.

Anna (surprised): Why, what's the matter with you?

P. G.: Oh, there's a lot the matter. I guess if everybody called *you* "plain" you'd be unhappy, too. Tell me something, will you, Sister? (Goes quickly over to where Anna is standing.) Why do you suppose everybody calls me "Plain"?

Anna: Oh, I don't know. They've just gotten into the habit of it, I guess. Everybody knows you are very interesting and dependable.

P. G.: Yes, but who cares about that when people don't find much fun or pleasure in me! (Pause.) At least, Jim doesn't seem to, and that's all that matters.

(*Al,* appearing at the doorway, hears Plain Geometry's last remark. Comes forward with mischievous smile.)

Al: Oh-ho! I see! So that is what has been making our little Plain go off on a tangent so easily of late. What's the matter with Jim—too much interested in little Miss Terious?

Anna: Now, Al, don't be such a tease!

P. G. (petulantly): There you go again, jumping at conclusions without sufficient data! Who said anything about Miss Terious!

Al (with a knowing air): Oh, Miss Terious is no unknown quantity to me; and I'll have you know that I can reason just as straight as you can, even if I don't Q. E. D. everything I say. I've drawn a conclusion or so myself on this subject.

P. G. (curiosity aroused): Why, what do you know about it?

Al: Never mind what I know about it. (Drops flippant and teasing manner.) Seriously, though, Plain, what makes you give up so easily just because they call you Plain? Why don't you develop more points of contact with people? I don't know any girl with a better "line" than you have, nor so many of them. Use them. That's the best formula I know.

P. G.: But *how* am I going to use them with Jim? That's the problem I wish you could help me solve.

Al: Well, why don't you have a party, invite the whole crowd and give Jim the time of his life?

P. G.: That's easy to say, but what could we do that he would enjoy?

Al: Do? Why there are lots of things we could do.

P. G.: Name just one of your "lots."

Al: One of them? Well, you could have Major Arc muster some of the loyal subjects and give a drill. You know Jim would get a big thrill out of that.

P. G. (skeptically): Yes, I suppose that would be good; but that's only *one* thing. What else could we do?

Al: Why not have a Square Dance?

P. G. (with growing interest): That's a good idea. I never thought of that.

Anna (who has been quietly listening to this conversation). Are you going to invite Miss Terious?

Al: Of course!

P. G.: Of course *not.*

Al: Oh, yes, you certainly want her, and I'll tell you why. I've been thinking you might have some good hard games. Miss Terious couldn't do anything with those and that would be your chance to shine.

Anna: I think Al is right, Plain. I'd invite Miss Terious, if I were you.

P. G.: Well, all right. Whom else are we going to invite?

Al: Oh, the loyal subjects of the curriculum and the students of ———— High School, I guess. Just the same old crowd.

Anna: Phil and Cal are home from college. Why don't you invite them?

Al: Phil and Cal who?

Anna: Why, Phil Osophy and Cal Q. Lus, of course.

P. G.: Oh, surely I'd like to have them. Do you suppose they will come?

Anna: Yes, I think so. When are we going to have the party?

P. G.: Well, this is Saturday. Let's have it two weeks from tonight. I'll see Major Arc today about the drill. And you'll have to help me, Al, with all the games and things.

Al: Count on me. I'm the best little trickster you ever saw, and it's pretty hard to see through my tricks. Ask any Freshman!

P. G.: You certainly are a peach, Al, for coming to my rescue.

Al: Oh! Why shouldn't I? It's all in the family. Besides —there's an unknown quantity in this proposition I'd like to find for myself.

CURTAIN

Act II

Scene: Same as Act I, with added decorations in geometrical patterns and designs.

(Curtain rises, disclosing Al and Anna dressed for the party, putting the finishing touches to the decorations.)

Anna: This has certainly been a busy two weeks. I don't believe you have ever worked so hard before in your life, have you, Al?

Al: Well, there's a reason!

(Enter P. G. hastily, dressed very prettily, and carrying a basket of flowers. These she puts on the table, arranging and surveying her work as she talks.)

P. G.: I've heard you make that remark at least sixteen times in the last two weeks, and I'm getting suspicious. I don't understand it.

Al: Don't let that worry you, little girl. You aren't the first one who hasn't seen through your cousin, Al!

(Bell.)

Anna: Oh! there's somebody already. You go to the door, Plain, and here, Al, help me.

(Exit P. G.)

Anna: Do you think everything looks all right? Stick that ruler in that drawer.

(Voices heard laughing and talking outside. Enter Cal, Phil, and Plain.)

Anna (advancing, smilingly shakes hands with Cal and Phil): I'm so glad that you were able to come. Have you met my cousin, Mr. Al Gebra, Mr. Cal Q. Lus and Mr. Phil Osophy?

(They shake hands and greet each other.)

Cal (to Al): I know your brother College Algebra very well. He and I have worked and been worked together for some time down at the university.

Al: Yes, I've heard him talk about you and (turning to Phil) I've heard about you too.

(Bell is heard. Exit Anna.)

P. G.: I guess that must be Jim and Miss Terious.

Phil: Who is Miss Terious?

Al: Oh! she is the cause of considerable interest at the present time in this locality.

(Voices heard laughing and talking. Enter Miss Terious, Anna and Jim. P. G. advances, shakes hands and greets Miss

Terious and then Jim. Al gives high sign of friendly greeting to Jim and it is returned.)

Anna: Miss Terious, may I present Mr. Phil Osophy, and Mr. Cal Q. Lus? (They acknowledge the introduction.) I believe you already know Mr. Al Gebra. (Al and Miss Terious greet each other and then enter upon a low conversation.)

P. G.: Mr. Jim Nasium, this is Mr. Phil Osophy and Mr. Cal Q. Lus. (They great each other.) And now I want you all to meet the other guests, the students of ———— High School.

(Boys smilingly salute and girls nod in friendly fashion to the audience.)

Jim: We just passed Major Arc out there. He was very much disturbed because one of the loyal subjects hadn't shown up. I wonder which one it was.

P. G.: I'll bet it was Trig. She is always getting her "Sines" mixed.

Phil: It wouldn't surprise me if it was Latin. She has so many cases, you never know whether she is going to keep a date or not.

Miss T. (coquettishly): It sounds as though it were rather difficult for you to be philosophical about her cases.

(All laugh.)

Major Arc (enters puffing, brandishing a blackboard compass. Salutes the hostess. All guests rise and return the salute): Good evening, everybody, you must pardon my breathlessness. I've been circumscribing myself about out there, and at last I have all the loyal subjects within a radius of my compass. With your permission, Miss Plain, I'd like to present my little drill to you and your guests before any of the subjects are dropped from the curriculum again.

P. G.: All right, Major, we are always ready to appreciate a good drill. Here, boys, help clear the floor to make room for the subjects. (Exit Major Arc.)

(Boys all scurry about and clear center of stage. The whole party then group themselves naturally around the sides. First strains of music are heard and Major Arc appears leading the subjects in the drill, "The Advance of the Loyal Subjects" adapted from "Parade of the Wooden Soldiers." See directions for drill at end of play.)

Drill.

(After drill much applause from members of party, together with approving comments from all. Boys move furniture back in place.)

Al: Say, I learned a good number stunt yesterday. I'd like to try it out on you. I'll bet nobody can guess how it's done.

Chorus: Let's hear it.

Jim: Shoot!

P. G.: When you said "Nobody," had you forgotten the talent of all these guests? (Makes sweeping gesture, showing that audience is included in party.)

Al (approaching from offstage and smiling at audience): No, of course, I hadn't forgotten. I want *everybody,* both down there and up here, to play it. (Makes gesture including first the audience and then the characters on the stage.) Come on, now. Here it is. Think of a number; multiply by 6; add 12; divide by 3; substract 2; divide by 2; substract the original number, add 9 and your answer is (slight pause) 10. (Turns toward audience with smile inviting response from them.) Am I right?

Chorus: Yes, how did you do it?

Miss T.: Why, I think you are wonderful. How did you know that was the answer we would all get?

Al: Oh! you did it. I didn't. You got the answer, didn't you?

Cal: Here's one for everybody (includes the audience, again): What is correct, 7 and 9 *is* 15 or 7 and 9 *are* 15?

(If possible, get response of *is* or *are* from audience. If it does not come, Miss T. says, "I think it should be 'are.' " If audience says: "7 and 9 are 16," skip to Phil's next lines.)

Anna (with puzzled look): That's funny. I always thought 7 and 9 are 16.

(General laugh.)

Phil: Since you seem to be in the mood for riddles, I have one to ask you. If a dozen cakes cost 26c, how many cakes can you get for a cent and a quarter?

(Pause for audience to consider.)

Jim: Not enough for one good bite.

Miss T.: Oh, dear, I never could work with fractions.

(Cal takes out small notebook and pencil and starts to figure.)

Al (puts hand in pocket and pulls out a cent and a quarter.

Drops cent from one hand to other, and then the quarter): Well, there's a cent and there's a quarter. Do you suppose I could get a dozen cakes with that?

Chorus: Oh! . . .

Cal: Does anyone know how to write 100 with four 9's? (Pause for audience to think.)

Miss T.: Nobody could do that. Why, there are only three figures in 100.

Al: How about $99\frac{9}{9}$?

Miss T.: You surely are good, Al.

Jim: I want a dance with the girl who can answer this question. (P. G. and Miss T. both look interested.) If you walk half way to the door, then half the remaining distance, then half what is left and so on, how long will it take you to get out of the room?

P. G.: Oh! that's easy. It's nothing but a geometric progression. You'd never get out.

Jim: Good work. (Bowing and offering his arm.) May I have the honor?

P. G.: With pleasure. Let's have a square dance. Come on, everybody. (Shove back the furniture.) Form parallel lines.

(See suggestion for dance at end of play.)

(They line up for a Virginia reel. P. G. and Jim, Al and Miss T., and Cal and Anna.)

Anna: Oh! we need another girl. Phil has no partner.

Phil: Need another girl with all these pretty girls down there! (Points to audience.) I should say not. Let me look them over. (Looks over audience as though deliberating on choice of a partner.) Will you be my partner, my pretty maid? (Girl addressed is one seated near front of auditorium. She knows beforehand that she is to be called upon. Virginia Reel follows. Jim gives undivided attention to P. G. and Al is plainly in the good grace of Miss T.)

Jim: Gee! Plain, I've had a good time. The Mathematics family takes the prize for knowing how to give a party.

Plain: I'm glad you liked it, Jim. It was a rather "plain" party and I thought plain things didn't appeal to you.

Jim: Oh, I say—That isn't worthy of your reputation for good logic. I'm sure it is what you mathematicians call a "fallacy." You know what I think of a certain girl who has the name of Plain.

P. G. (coyly): I wonder! What?

Jim (ardently): Charming!

P. G. (breathlessly): O-oh!—but look, Jim (calling his attention to the other group), Al and Miss T. are simply *engrossed* in each other.—Don't you mind?

Jim: Mind! Really, Plain, I didn't know you could be so obtuse! Why those two are just meant for each other. Al always has been searching for the unknown; now he has found her. As for me—well—I'm sure I quite prefer a "plain" future to a "mysterious" one!

(Al and Miss T., who have been talking in an aside, approach to say their farewells.)

Miss T. (holds out her hand to Jim): Goodnight, Jim. (And then to P. G.). And, thank you, Plain, for inviting me to your lovely party.

Al: It was a good party, Plain; and best of all the "problem" is solved.

P. G.: "Problem?" What do you mean, Al?

Al: Why, don't you see? What looked like the "eternal triangle" has been "completed" and has now become the "perfect square."

CURTAIN

The Square Dance

As a suggestion for the square dance, "If All the World Were Paper" is mentioned. The music and dance instructions by Cecil J. Sharp and George Butterworth may be obtained from The H. W. Gray Company, New York City.

Advance of the Loyal Subjects

Character group dance adapted from "Parade of the Wooden Soldiers" as given in the "Chauve Souris" by Louis H. Chalif.

The music is "The Parade of the Wooden Soldiers" by Leon Jessel, published by the E. B. Marks Company, 225 W. 46th Street, New York City. Phonograph records may also be had.

The dancers who undertake to impersonate wooden characters will have to concentrate upon the idea of rigidity as to the body and absolute expressionlessness of countenance. The utmost seriousness must be maintained throughout the dance.

The subjects wear stiff white duck trousers; dark blue ging-

ham jackets with red cuffs and collars, closed high and straight to the chin. The jacket front has two rows of gilt buttons. Dark blue paper military caps are worn. Yard sticks are carried. The Major wears a similar costume except that a white band goes over his right shoulder and passes under his left arm. He also wears a large medal on his chest. A blackboard compass is in his right hand. They do not appear until the introductory music is played. ————4 measures

Figure I

The soldiers enter in single file from the center of right of stage (see diagram 1), being led by No. 1, the major, and the drummer, No. 9, being at the other end of the line. All carry the gun in left hand, hold it perpendicularly and rather low; their forearms are bent forward at a right angle. They are as close together as they can be and walk. Their steps are very short, quick marching steps (2 steps to a count), with the weight on the heels, although the toes are not lifted, and with knees held stiff and straight.

They march thus for 14 counts, starting left face, then all make a quarter-turn to right to face audience (count 15,) mark time in place for 2 steps (count 16) . . . . 8 meas.

All march forward toward audience with the same short steps for 14 counts (see diagram 2), make a half-turn to right to face rear of stage (count 15), mark time in place (count 16) . . . . 8 meas.

March toward rear of stage (see diagram 3) for 14 counts, make a half-turn to right to face audience (count 15), mark time in place (count 16) . . . . 8 meas.

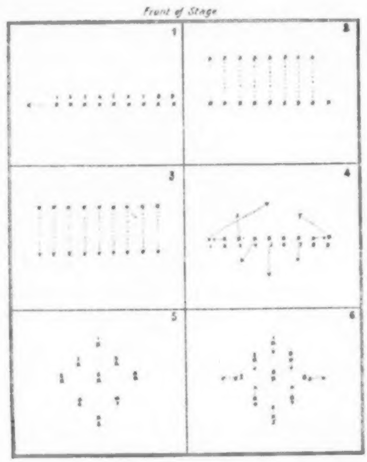
They will now make a diamond-shaped formation (see diagrams 4 and 5). Those who move backwards will march backwards, without turning around. They use the ordinary marching steps for 7 counts and rest on count 8. No. 6 marks time while edging to the center. . . . . 4 meas.

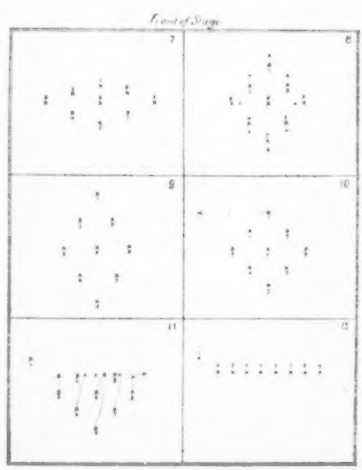
28 meas.

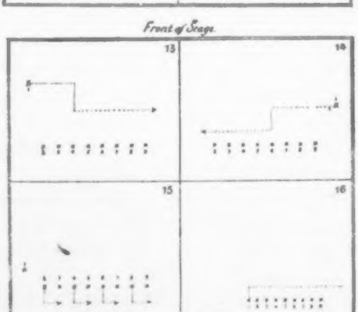
Figure II

All face right with 3 quick, short steps, thus: step right face, turning to face right (count 1), step left face beside right face (count and), step on right face in place (count 2), pause (count and). . . . 1 meas.

During this measure, all will continue to face right while moving in various directions. No. 6 will mark time in place. The 3 nearest the audience (Nos. 3, 1 and 9) step sideward to right toward rear of stage, thus: step right face to right (count 1), step left face beside right face (count and), step right face in place (count 2), rest (count and). At the same time the 3 at the rear (Nos. 4, 5 and 7) move to left toward audience with the same steps, starting left face, and the 2 at the sides (Nos. 2 and 8) do the same steps outward toward the sides of the stage, No. 8


Front of Stage
1
2
3
4
5
6


Front of Stage
7
8
9
10
11
12


Front of Stage
13
14
15
16

stepping forward and No. 2 backwards (see diagram 6 for the direction of movement and No. 7 for the place arrived at). Thus it is seen that the 2 groups of 3 are moving toward the center of the group, which direction we will in the future call "in," while Nos. 8 and 2 are moving "out." The steps will always be 2 short steps, as just described. 1 meas.

Everyone does left face (a quarter-turn to left to face audience), using 3 steps as before. . . . . 1 meas.

Everyone continuing to face audience, the 2 groups of 3 step "out," those in front stepping forward and those in the rear backwards, while Nos. 2 and 8 step "in," moving sideward. No. 6 should always mark time when the others move from their places, but should change his direction of facing whenever they do. They should now be in the original position of diagram 6 again . . . . 1 meas.

Repeat all, but starting facing left, thus:

Everyone face left . . . . 1 meas.

The groups of 3 step in and Nos. 8 and 2 out . . . . 1 meas.

Everyone face right . . . . 1 meas.

The groups of 3 step out and Nos. 8 and 2 in, finishing this part of the figure . . . . 1 meas.

While facing the audience, as they now are, the 2 groups of 3 step "out" and Nos. 8 and 2 step "in." See diagram 8 for direction of motion and diagram 9 for position arrived at . . . . 1 meas.

The 2 groups of 3 step in; Nos. 8 and 2 step out, returning to original position . . . . 1 meas.

Everyone turns half-way around to left to face rear of stage (left about face), taking 7 little steps in place to do so (4 counts) . . 2 meas.

While all face rear of stage the 2 groups of 3 step out and Nos. 8 and 2 step in . . . . 1 meas.

The 2 groups of 3 step in and Nos. 8 and 2 step out . . . . 1 meas.

All do left about face, taking 7 little steps to do it . . . . 2 meas.

16 meas.

Interlude

While the company marks time inconspicuously, facing forward, the major (No. 1) moves sideward toward left of stage, while facing forward, with little steps on the heels (2 to a count always).

When he has reached a position a little to left of the company (on count 7 or 8) he turns to face right of stage and makes a signal to the girls by extending his sword forward, holding it perpendicularly upward. In the meantime the drummer (No. 9) beats her drum with stiff, wooden movements of her arms, without actually touching the drum with her drum-sticks, thus: Move right arm down and left arm up (count 1), reverse the position of the arms (count and), reverse the position again count 2), repeat (count and), repeat 3 times more . . . 4 meas.

Figure III

Answering the signal of their major, the company form one straight line again, as follows: the drummer moves sideward to right a little way,

while all the others move forward to be in a straight line with her, i.e., ther "dress to the right." Each should go to her original place in the line. They take marching steps in the usual rhythm (16 counts). (See diagrams 11 and 12) . . . . 8 meas.

The major does not move during the above 8 measures until at the end he moves his right arm sideward at shoulder level, still holding the sword perpendicularly.

At this signal the girls march backwards, while facing audience, with the usual marching steps, while the major stands still (16 counts). 8 meas.

16 meas.

Figure IV

The major marches with very short step across in front of the company toward right of stage (12 counts), he makes a quarter-turn to right and marches toward the company (4 counts), he makes a quarter-turn to left and continues marching in original direction and finishes at the right, facing left of stage (16 counts) (see diagram 13). During this time he carries his sword perpendicularly at the side and has his left arm bent forward. Meanwhile the company marks time while doing "Right Dress" as follows: the girl at the left (No. 2) turns her head sharply to right (count 1), and holds the position during count 2, 3, 4 and continues holding it for the remainder of 16 measures. On count 5 (the beginning of the 3d measure) No. 3 turns her head to right and holds it thus for the remainder of 16 measures and so on down the line, each girl having 4 counts or 2 measures for her turning of the head (32 counts). But on the very last count all turn their heads forward again, as if the command "Front" had been given. . . . 16 meas.

Interlude

The major raises his arm forward with sword held perpendicularly as a signal, whereupon the company does "Present Arms" as follows: with right hand take hold of the gun near left shoulder (count 1, 2), hold it out forward perpendicularly, held by both hands (count 3, 4), return the gun to its former place at the left side, still keeping hold of it with right hand (count 5, 6), bring right hand down to its original position of being bent forward from the elbow (count 7, 8). . . 4 meas.

Repeat . . . . 4 meas.

8 meas.

Figure V

While the music stops for a moment the girls say "Hurrah" 3 times in gruff, wooden voices and running the 2 syllables together to make it sound like one. The major now retraces his steps to the left of stage. He marches toward left (4 counts), makes a quarter-turn to left and marches toward the company (4 counts); while standing with heels together, facing the company, he almost loses his balance; he rocks forward as if about to fall (4 counts), rocks backwards (2 counts) and rocks for-

ward again, recovering his balance (2 counts). . . . . . . 8 meas.

He makes a quarter-turn to left and proceeds to left of stage (4 counts); having gone a little beyond the end of the line and being a little in front of it, he makes a half-turn to face right of stage (count 15), he opens right arm forward and to side, all in one gesture that is a signal (count 16) (see diagram 14). . . . . 8 meas.

Meanwhile the company has been marking time.

The music for Figure V should be repeated.

The odd-numbered girls stand still while the even-numbered ones march backwards toward rear of stage, starting left face (8 counts), march sideward to right until each is directly behind the girl who was at her right, so that they are in 2 lines (8 counts) (see diagram 15). They then march sideward to left, retracing their steps (8 counts), and march forward to their former places in line (8 counts). . . . . 16 meas.

32 meas.

Interlude

The company slowly faces left, while marking time, then all except the drummer move backwards, to be as close together as possible (7 counts), rest (count 8). Meanwhile the major slowly makes a quarter-turn to left to face audience with marching steps (4 counts), then marches backwards to the end of the line, and finishes facing the audience (4 counts). 4 meas.

Figure VI

Being led by the major the company starts to march off the stage at the right in single file. To begin this the major marches forward toward audience for 8 counts, then makes a quarter-turn to right and marches toward right, while the company follows him in single file, marching as close together as possible, and making square turns at the corners. They continue marching until the catastrophe occurs. . . . . 14 meas.

All at once it seems as if an unseen, gigantic hand had carelessly brushed against the wooden subjects. They reel backwards, beginning with the major who bumps against the one behind him and so on. But this movement is soon arrested and counteracted by swinging in the opposite direction; all soon begin to sway back and forth, each rocking as she happens to, until on the last note of the music they come to rest, leaning this way and that, being braced against each other. At least the supreme calamity of an ignominious fall to the floor has been averted.

Our gallant subjects have not been overthrown. . . . . . 8 meas.

22 meas.

HAS ALGEBRA CERTAIN REAL VALUES FOR THE HIGH SCHOOL STUDENT OF TO-DAY?

A Radio Talk

By Professor WINONA PERRY

University of Nebraska

Why are nearly all of the high school students of to-day asked to include at least one course in algebra on a year's program of study? Is it that one of the uses of algebra is to enable the student to be better prepared to undertake the study of the sciences? A rather large number of instructors in certain science subjects have indicated, on carefully prepared questionnaires, that certain topics in algebra are considered fundamental to the most effective learning of their particular subject. Of immediate importance to us was the series of statements revealing that the easier parts (rather than the long, involved, and difficult aspects) of certain topics were the ones more frequently used in other subjects. These topics differed also in the number used and the degree to which they were considered essential by the different sciences—the physical sciences (as physics and chemistry) making more and heavier demands upon them than the social sciences (as history, sociology, and psychology). One value of algebra to the high school student of to-day is, then, that certain topics are necessary as the basis for studying certain scientific subjects—more especially, the physical sciences. Largely because of this fact are professional and technical schools requiring all of their entering students to have taken and passed creditably certain specified courses in high school algebra. If, however, a student does not wish to take one or more courses in science, nor to enter a professional school (including courses such as those offered in medical or engineering schools)—will such a student have occasion to use his knowledge of algebra in his general reading or in study of other courses? As the Encyclopedia Britannica meets some of our reading needs, it was read with the specific purpose of noting to what extent readers would be unable to grasp the meaning and significance

of those articles, had those readers not mastered certain topics in algebra. It was found that two topics—the graph and the formula—were essential to read the Encyclopedia Britannica with ease and understanding, but that the other topics bore little if any relation to this type of reading need of the student or of the adult. Had newspapers or magazines been studied similarly, we should probably have had a more accurate conception of the extent to which certain topics in algebra are essential to the reading of adults, as well as of boys and girls now enrolled in high schools. Some real values, then, of algebra to the high school students of to-day come from their need to understand its phraseology and to be able to use their knowledge of it as a tool in some of the courses in sciences, to present credits in it in order to be admitted as a student to certain professional schools, and to a lesser degree to be able to read understandingly articles in the Encyclopedia Britannica, or other books of reference.

Are the boys and girls in our high schools capable of deriving much benefit from studying algebra as it is being taught at the present time? How able are those students who are entering our high schools now? In general we are finding that our boys and girls are among the more able people in a community; *i.e.,* if everyone were placed according to his ability to learn, to adapt to, and to understand situations, acting accordingly with ease and rapidity, then our high school boys and girls would probably be, on the average, among those grouped in the upper half of this arrangement. Frequently the boy or girl whose ability indicates that he would belong among those within the lower half of the general population, drops out of school before the beginning of the senior year. To these latter boys and girls we are recommending that they elect a different type of mathematics course, or that they study algebra during their second or third year in high school. All other boys and girls may choose to study algebra during their first year in high school with the assurance that they will probably grasp the principles of algebra with success, and with confidence in their own ability to do so.

Our question also asked if our boys and girls would profit from studying algebra *as it is being taught to-day.* We are watching our students with exceeding care during the times

when they are actually learning the meaning of algebraic processes and learning 'how to do' the problems, noting the results of the learning of our students measured by tests which have been most carefully prepared. All to what purpose? So that we may discover just what are the student's difficulties? Why does a boy or girl hesitate to start a problem? Why, after he has started to solve it, does he or she hesitate, sometimes frown or become discouraged? Why does he sometimes proceed vigorously and quickly and with real satisfaction? What can we teachers do to guide all of the students to reach a successful solution when a problem has baffled them and they have stopped "to think it out"? We are finding more and more how to prophesy just what difficulties students are going to find, and to plan a series of problems which will cause the student to face those difficulties one at a time, then gradually to learn how to attack the more complex problems with greater ease and satisfaction.

Furthermore we try to arouse in them pleasure in increasing the number of problems solved and accuracy in solving. The following is an example of such a practice test:

> EXCELLENT ninth grade pupils are able to solve 20 of these equations in 5 minutes, after three or four trials; GOOD pupils can solve 16 of them, and FAIR pupils can solve 10. How many of these equations can you solve?

The 30 equations of actual practice material are followed by the question:

> Did you score excellent, good, or fair on this trial?

Boys and girls like this and work very intently and eagerly. They like to beat records, and they are willing to try again and again to see if they can be able to surpass a record "this time." We are so planning our methods of teaching that we can take advantage of this very human desire—in fact, it is stronger than a desire, for it is really an innate power that is driving us toward reaching a self-set goal. Our students are not urged to surpass another student in the same class, but they are urged to establish their own record and then to see, with a certain amount of practice in a given time, to what extent they can "beat their own records."

We have a purpose here that goes even beyond the immediate capture of the student's interest in his own improvement, al-

though that in itself is a worthy purpose. We have found that if the student is interested in the actual learning process, he will master what he is learning more quickly and remember it longer. Consequently as students do learn much more economically and effectively if they are interested while they are learning in the results of their learning, and, reciprocally, their learning is more permanent, our practice in algebra tests is so planned that they will arouse and increase the interest of our boys and girls in their learning of principles and processes.

What, then, are the real values to our students who have secured a high degree of mastery of algebraic skills? Algebra is a necessary tool in the learning of the physical sciences; it is required for entrance into certain professional and technical schools; certain parts of it increase our understanding and enjoyment of general reading. Moreover, as the more able and purposeful student gradually becomes aware of the fact that his difficulties in algebra are lessening and that his pile of successes, so to speak, is mounting higher and higher, his interest increases and likewise his satisfaction in meeting those situations successfully. An authority in this field, E. L. Thorndike, frankly states:

> The chief appeal of algebra has been, and probably always will be, to the love of thought for thought's sake by those who can play the game of thought well.

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MINUTES OF THE EIGHTH ANNUAL MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, DALLAS, TEXAS

(*A*) Joint Meeting of the Executive Committee and Local Committee, Friday Evening Session, February 25, 1927, Hilton Hotel, Dallas, Texas.

The meeting was called to order by the President, Marie Gugle, Assistant Superintendent of Schools, Columbus, Ohio. President Gugle appointed J. O. Hassler, Beulah Baker, and W. O. Franks as tellers to count the ballots for the 1927 election of officers. The report of the tellers was as follows:

Number of ballots cast—104. Officers elected: President, Marie Gugle; Vice-President, C. M. Austin; Secretary-Treasurer, J. A. Foberg; Members of Executive Committee for three years, Vera Sanford, and William Betz. The tabulated vote follows:

For President:		*For Vice-President:*	
Marie Gugle	82	C. M. Austin	75
W. A. Austin	22	C. N. Stokes	28
For Secretary-Treasurer:		*For Member of Exec. Com.:*	
J. A. Foberg	72	Vera Sanford	69
E. W. Schreiber	28	H. C. Christofferson	33
		William Betz	62
		T. J. Johnson	49

The Treasurer's Report was read and accepted. The Report of C. M. Austin, Chairman of the First Yearbook Committee, was read and accepted. The Report of John R. Clark, Editor of the MATHEMATICS TEACHER, was read and accepted. A motion was made by John R. Clark and seconded by Harry English that W. D. Reeve be appointed Co-Editor and Business Manager of the MATHEMATICS TEACHER. Carried. The Report of President Gugle was read and accepted. A motion was made by C. M. Austin and seconded by W. D. Reeve that meetings of the Council be held on Friday and Saturday in the future. Carried. A motion was made by F. C. Touton and seconded by Harry English that a committee be appointed by the President to investigate and determine the validity of all claims against the Council and report to the President. Carried. A motion was made by W. D. Reeve and seconded by F. C. Touton that a

maximum of \$500 be used at the discretion of the Executive Committee for the extension of the work of the Council. Carried. A motion was made by F. C. Touton and seconded by W. D. Reeve that a committee be appointed by the President to revise the Constitution of the Council. Carried. A motion was made that two (2) sets of each issue of the MATHEMATICS TEACHER and two (2) copies of each Yearbook be obtained and placed on file as the property of the Council. Carried. A motion was made that the President appoint a committee to prepare a complete historical account of the Council. Carried. A motion was made and carried that \$25 be paid to C. M. Austin and \$100 to President Gugle to cover expenses. A motion was made and carried that President Gugle be paid \$33.10 to cover incidental expense. A motion was made and carried that 6,000 copies of the Second Yearbook be printed.

(Note: The above minutes were taken in brief by C. M. Austin of Oak Park, Illinois. They were transcribed in their present form by Edwin W. Schreiber of Maywood, Illinois, who was appointed by President Gugle as Secretary for the Dallas Meeting.)

(*B*) General Session, Saturday morning, February 26, 1927, Adolphus Hotel.

The Eighth Annual Meeting of the National Council of Teachers of Mathematics was opened by President Gugle at 9.10 A.M. The meeting was held on the fifteenth floor of the Adolphus Hotel in very attractive quarters known as Bambooland West. President Gugle announced that the Second Yearbook was available and she hoped that all present would purchase a copy from the Editor, W. D. Reeve. A motion was made by C. M. Austin and seconded by E. H. Taylor that the minutes of the Washington Meeting of the Council as printed in the MATHEMATICS TEACHER (April, 1926) be approved. Carried. The Treasurer's Report was read by Edwin W. Schreiber and on motion was accepted as read. C. M. Austin made his report as Chairman of the First Yearbook Committee. At the suggestion of J. R. Clark a rising vote of thanks was extended Mr. Austin for his labors in connection with our new venture in yearbooks. John R. Clark, Editor of the MATHEMATICS TEACHER, gave a brief report on the present status of the "Teacher." J. O. Hassler, Chairman of the Teller's Committee, read the report

of the election of officers of the Council as determined by the official ballot. W. D. Reeve gave a report on editing the Second Yearbook. The general topic is: "Curriculum Problems in the Teaching of Mathematics." The Council is very fortunate in having the distribution of the Second Yearbook effected through the Bureau of Publications, Teachers College, Columbia University. The price is $1.25 for the paper edition and $1.75 for the bound edition.

The formal program of the morning began with a paper by Dr. John R. Clark on "Investigations in the Teaching of Plane Geometry." Considerable discussion followed in which the following took part: Mr. Cameron of Chester, Pa.; C. M. Austin of Oak Park, Ill.; J. F. Howard of San Antonio, Texas; F. C. Touton of Los Angeles, Cal.; Edwin W. Schreiber of Maywood, Ill.

William A. Austin of Venice, Cal., was unable to be present at the Dallas Meeting but his paper on "The Laboratory Method of Teaching Geometry" was read by Frank C. Touton, who also discussed the paper.

"Individual versus Group Instruction in Ninth Grade Algebra" was the title of an interesting paper presented by C. B. Marquand, West High School, Columbus, Ohio. J. M. Bledsoe of Commerce, Texas, discussed this paper.

Leonard D. Haertter, John Burroughs School, Clayton, Missouri, gave an enthusiastic paper entitled "Efficiency of Instruction in Large and Small Classes." Mary S. Sabin of Denver, Colo., led in the discussion of Mr. Haertter's paper.

(*C*) Joint Meeting of the Executive and the Local Committee, Saturday Noon Session, February 26, 1927, Adolphus Hotel.

While a delicious luncheon was being served and enjoyed the following items of business were transacted: President Gugle appointed as the members of the Committee to revise the Constitution, Professor H. E. Slaught, C. M. Austin, and Edwin W. Schreiber. This Committee was also authorized to investigate incorporation proceedings with power to act. President Gugle appointed W. D. Reeve and Harry English as members of a Finance Committee to interview J. A. Foberg and determine the present financial status of the Council. A motion was made by Harry English and seconded by E. W. Schreiber that W. D. Reeve be made Editor of the Third Yearbook. Carried. A

motion was made by F. C. Touton and seconded by E. W. Schreiber that future yearbooks be edited from the same office as the MATHEMATICS TEACHER and that if additional editorial help is needed the Editors of the MATHEMATICS TEACHER may secure same if approved by the Executive Committee. Carried. A motion was made by F. C. Touton and seconded by W. D. Reeve that 500 or more copies of the Second Yearbook be bound and sold for $1.75 per copy. Carried.

(*D*) General Session, Saturday afternoon, Adolphus Hotel.

The afternoon session opened with a paper by Elsie Parker Johnson, Oak Park High School, Oak Park, Illinois, entitled: "How to Make the Concept of the Locus Real." Many interesting illustrations of locus problems were presented in this paper. Professor H. E. Slaught, University of Chicago, gave an excellent paper on "The Romance of the Number System," which was well received. "Some of Euclid's Algebra" was ably presented by George W. Evans of Houston, Texas.

(*E*) Annual Banquet, Saturday evening, Adolphus Hotel.

The Annual Banquet of the Council was held at 6.00 P.M. in the Adolphus Hotel. Miss Elizabeth Dice, North Dallas High School, had charge of arrangements and reservations for the Banquet. After a delicious repast President Gugle introduced the guests of honor, Superintendent and Mrs. N. R. Crozier of Dallas, Texas. Superintendent Crozier responded with an address of cordial welcome. President Gugle then called upon Miss Dice who informed us that forty-two Dallas teachers had assisted in the local arrangements for entertaining the Council. Dr. W. D. Reeve, Teachers College, Columbia University, formally presented the Second Yearbook. After telling of some of the trials and tribulations of editing a yearbook, Professor Reeve outlined the contents of the Second Yearbook. Part I is devoted to "Curriculum Problems in Arithmetic"; Part II concords "Junior High School Mathematics"; and Part III deals with "Senior High School Mathematics." Miss Rose Hammond, Fair Avenue School, Columbus, Ohio, discussed Part I, dealing with problems of Arithmetic. Dr. Reeve discussed Part II on Junior High School Mathematics. Senior High School Mathematics was discussed by Edwin W. Schreiber, Proviso Township High School, Maywood, Illinois. Among those who took part in the general "free for all" discussion were the fol-

lowing: Dr. E. H. Taylor, Charleston, Ill.; Supt. Crozier, Dallas, Texas; Prof. H. E. Slaught, University of Chicago; C. M. Austin, Oak Park, Ill.; Prof. Raleigh Schorling, University of Michigan; Prof. Frank C. Touton, Los Angeles, Cal.; Prof. H. O. Hassler, University of Oklahoma; Prof. Philips, Emporia, Kansas; Elsie Parker Johnson, Oak Park, Ill. President Gugle adjourned the evening session with "thank-yous" to all who had made the Eighth Annual Meeting such a splendid success.

EDWIN W. SCHREIBER,
Secretary of the Dallas Meeting.

NEW BOOKS

Z Fahl und Raum (Number and Space). By STUDIENRAT DR. F. MALSCH with the collaboration of STUDIENRAT PROFESSOR DR. E. MAEY and STUDIENRAT H. SCHWERDT. Published by Quelle und Meyer, Leipzig, 1925–6.

This is a series of texts suitable for use in the secondary schools of Germany from the seventh grade through the thirteenth.[1] For these seven grades this series provides seven books (and an eighth dealing with the calculus for certain schools). But this does not mean that the books are to be used in order, one in each grade. The intention is rather to maintain the continental tradition of teaching geometry and algebra in continuous parallel courses over a period of several years, emphasizing wherever desirable the numerous pertinent applications of one subject to the other, but at the same time keeping the two essentially distinct and intact, and so giving an appreciation of each by itself as well as an appreciation of each in relation to the other. The title of the series, Number and Space, suggests both aspects.

Three of these books are entitled Arithmetic and Algebra, Parts I, II, and III; three more are called Geometry, Parts I, II, and III; and then there are two dealing with analytic geometry and the calculus. Ordinarily Geometry, Part I, would be begun in the seventh grade and continued in the eighth grade along with Part I of the Arithmetic and Algebra; and Algebra, Part II, would be completed before Part II of the Geometry. But every latitude is accorded the teacher to arrange the course of study as he sees fit. The books are not long, averaging about 125 pages, a large proportion of which are devoted to exercises. This is accomplished by restricting the expository material and motivation to brief but adequate compass, in full confidence that the individual teachers will amplify it according to the needs of their pupils—a confidence which is justified by the professional training and knowledge of subject matter possessed by these

[1] The grades have been numbered to correspond with the usual practice in the United States. Thus the seventh grade comprises pupils who have completed six years of schooling and are from twelve to thirteen years of age; the thirteenth grade is our first year of college.

teachers. For example, thirteen of the twenty-seven pages dealing with negative numbers and parentheses are devoted to exercises.

One should not judge from this that the presentation of the material ignores the psychological approach and is not alive to the newer trends of educational thinking. The author's preface has a very familiar ring: ". . . wholly in accord with the spirit of the National Committee's Report, which, in response to the recent Youth Movement in Germany, would relate mathematical instruction more definitely than heretofore to the practical needs and interests of the students and render it more vital in terms of their environment and everyday life; would supplant dogmatic methods of instruction by an inductive method based on observation and learning by doing; and would insure that the outcomes of this instruction should be not merely the techniques and disciplines necessary in the equipment of the young builders of the future German state, but should include also a real appreciation of the relation of mathematics to other arts and sciences, as well as of the part it has played in the cultural development of the race. . . ."

These texts clearly mirror the social and economic upheaval in Germany and are but one example of many adjustments to a new order. Whereas before the war almost all graduates of the gymnasium went to the university, now over fifty percent of such graduates must go to work. Under such circumstances it is recognized that secondary education cannot remain divorced from everyday life, but must be immediately related to the world in which the students live and which they will so soon have a part in shaping. Many of the exercises have a practical bearing, and commercial arithmetic and commercial algebra have a significant amount of time allotted to them, with problems drawn from Rhein and Ruhr.

It is also recognized that such a program can be carried too far and result in the turning out of narrow specialists trained for the immediate present but incapable of adapting themselves readily to the changes of a more remote future. Ample provision is made throughout the series, therefore, that the student shall grasp the cultural significance of mathematics, and come to appreciate its philosophical implications and its historical development not simply as isolated items, but as in-

timately bound up in the growth of human civilization. Not only are there reproduced pages of mediæval German texts, but problem material from these sources is scattered through the series to indicate the content and mode of thought of earlier times. The evident desire that students in the secondary schools shall learn to see life whole is reflected not only in this humanistic corrective for the practical work in mathematics, but in a much larger way in the establishment of a new type of "gymnasium"—the Europäistiches—which in motive at least has an earlier parallel in this country in the founding of the English high schools to meet the needs of those for whom the academies offered no proper curriculum.

A number of details as to the content and method of these texts are of interest to us. The equation is introduced in the seventh grade through puzzle situations such as $?/4 = 11/13$. Positive and negative numbers are called Relative Numbers where many of our texts speak of Directed Numbers. There is very little work on factoring. The presentation of rules, principles, and abstractions is regularly preceded by several numerical examples to give point to the generalization which follows. Many of the problems propose a definite situation from which the student himself is expected to frame the appropriate question and answer it. The treatment of non-mathematical graphs is very elaborate and there are numerous tables of statistics among the exercises for graphic representation. The graphic representation of equations of the first degree in two variables is taken up before simultaneous equations. But the graphic work, though prominent, retains always its proper place and does not attempt to usurp the entire domain of algebra. This is attested by the treatment of simultaneous equations, which begins with the method of substitution, then proceeds to the method of addition and subtraction, and concludes with a graphic treatment which is almost too meagre at this point. The desirability of scrutinizing answers to see that they are approximately correct, and of checking them accurately by means of substitution, receives no mention in these texts. There is an interesting philosophical note on the extension of the meaning of "exponent" to include fractional and negative exponents, and another on the nature of irrational numbers in terms of a division of the totality of rationals. But this treatment of frac-

tional exponents follows after the treatment of radicals and is wholly secondary, instead of being made the principal method for which the notation involving radicals is simply an interesting and historic alternative, necessary only because others use it. And again, "imaginary" numbers are introduced before the solution of the general quadratic equation, where we should choose to solve the quadratic first and let the need for the complex numbers grow out of that treatment. It would seem a fair inference that most schools would complete the work on exponents and quadratics before the end of the ninth grade, leaving time in that grade for the treatment of logarithms and the slide rule. The later work in algebra for the tenth, eleventh, and twelfth grades completes the usual elementary treatment of series and the binomial theorem, and includes an important section devoted to questions in advanced commercial arithmetic which involve compound interest and are easily handled by algebraic methods. The usual topics of advanced algebra, commonly taught in colleges in this country, conclude the work in arithmetic and algebra. Strangely enough the explicit discussion of a linear-quadratic pair of simultaneous equations seems not to be treated until the very last stages of the work in algebra, and then in connection with graphic methods of solving equations of higher degree.

The earlier work in geometry is based on drawing and mensuration, and is intended for grades seven and eight. Three-dimensional material is considered along with two-dimensional material, and this emphasis on three dimensions persists throughout the course. There is no attempt at this stage to give a close logical treatment of geometric propositions, but rather through abundant experience to induce and enunciate the basic propositions and facts of geometry. Much use is made of symmetry in investigating geometric situations. In succeeding stages of the work in geometry no effort is made to rebuild this framework of geometric propositions on a deductive basis, but rather to accept as sufficiently established the mass of propositions resulting from the previous emphasis on drawing, and to proceed with increasing rigor to theorems on similarity and proportion and harmonic division. But nowhere does one find the severely deductive treatment which we are accustomed to associate with demonstrative geometry. Instead, the student is introduced to

numerous applications of geometry such as the pantograph, the plane table, the representation of solids by orthogonal projections, and the elements of descriptive geometry. The elements of surveying are taught in connection with the instruction in trigonometry. In the final stages of geometry (designed for the eleventh, twelfth, and thirteenth grades) there is no hesitation to adopt a new sequence of topics or to draw upon the easier portions of subjects ordinarily reserved for university study. The geometry of the sphere is linked with the study of map projections; plane trigonometry is extended to include so much of surveying as is comprised in the triangulation of a network; spherical trigonometry is applied to the astronomical triangle; and the deductive treatment of lines and planes in space is associated with the elements of projective geometry, including cross-ratio and the properties of the complete quadrilateral and quadrangle. Not all schools will choose to cover all these topics. There is wide latitude for selection, and the choice will be influenced somewhat by the amount of time it is desired to reserve for analytic geometry and the infinitesimal calculus in the twelfth and thirteenth grades.

Despite the many differences between this series and the older German texts, it nevertheless holds fast to the traditional ideal that the mathematical instruction in each curriculum should be comprehensive and thorough, and should be regarded as a continuous whole from start to finish. There is a lesson here for us which we cannot afford to ignore. If our educational aims and ideals have been in the immediate past so different from those of other countries as to warrant—in some measure at least—our proceeding in disregard of their experience, the signs of the times would seem now to point to a growing community of interest—economic, social, and cultural—which makes everyone else's problem akin to ours, and his solution worthy of our most careful consideration.

RALPH BEATLEY

Thinking About Thinking. By CASSIUS J. KEYSER. E. P. Dutton & Company, 1926. Pp. 91. Price $1.00.

Professor Keyser's essay *Thinking About Thinking* is a discussion of a type of thought which the author calls "autonomous" or "postulational." This type of thinking is auton-

omous or self-governing in the sense that its assumptions and conclusions make it logically complete: it is postulational because it rests on hypotheses assumed as a basis of further work. After a preliminary description of this type of thinking, the author discusses its importance and the fields in which it may be employed. Finally, he stresses the value of the careful scrutiny of postulates and the discarding of those that are not true.

Even if Euclid's *Elements* had not been mentioned as the earliest great example of postulational thinking, many readers would have supplied this point for themselves. These are the people to whom the study of geometry is an adventure in thinking, not the mere following of another's thought: the people who read into it a deeper significance than the study of lines and triangles. To such a group, this essay represents the most cogent plea for the teaching of geometry in present day literature, although it is highly probable that the author did not have this specific purpose in mind but that he was concerned with the application of postulational thinking beyond the class room. Yet it can safely be said that Professor Keyser is not likely to be disappointed if the effect of his essay is to make teachers of geometry more conscious of the possibilities of the type of thinking which they are trying to develop. Perhaps if we were really aware of the importance of postulational thinking, our work in geometry would have greater results.

VERA SANFORD

THE LINCOLN SCHOOL

A Graphic Analysis

of

Modern Junior Mathematics

By MARIE GUGLE